# Problem 1:

Common Tangent Circle arc of two given circle

## Input

All points are represented in this format : point(float x,float y)

Given points : S,E,L,R

These points should make 4 arcs of W smooth. Otherwise,it is bad input.

## Objectives

Two circle arc tangent to circle S and circle E

## Output

Circle arc A and Circle arc B which are tangent to circle S and circle E

Circle center and radius of common tangent circle A and common tangent circle B.

## Validation

Calculate the distance between circle S and circle A to examine whether they are tangent.

Do it again with circle E and circle A.

Then use the same method to examine circle B.

## Outline of your approach

Use vector to calculate the intersection point of two tangent line and the tangent point.

We can assume intersection point O is L + t SL’, then every unknown variable can be represented by t

Finally, we will get a equation with only one unknown variable t. So we can solve it and get t.

## Results

It works well with valid input.

It will fail when input are not valid because we can’t find 4 arcs smoothly connected with bad input.

## Details

Assume O is intersection point of two tangent line

T = SL’ // SL rotate PI /2 towards O

O = L + tT

rb is radius of circle E

EO \* EO = t\*t + rb \* rb

(EL + tT) \* (EL + tT) = t\*t + rb \* rb

Dot(EL , EL) + t\*t\*dot(T,T) + 2 \* t \*dot(T,EL) = t\*t + rb \* rb

t = ((rb \* rb - dot(EL , EL))) / (2 \* dot(EL, T))

Pseudocode:

Use the method above to get the value of t

SL ← vector from L to S

LO ← Unit vector perpendicular to SL

O = L + t \* LO

EO ← vector from E to O

ETan ← unit vector of EO rotated to tangent Point P, rotate angle is acos (r / length of EO )

r ← radius of circle E

P = E + r \* ETan.

DrawArc(L,O,P)

Use the same method to get another arc

## References

*Titles, Authors, and links to web sites or papers that have helped you. Add a short sentence explaining the merit of each.*

## Further research

*What else would you want to try to improve, generalize, or apply your solution.*

# Problem 2: Medial Axis

## Input

All points are represented in this format : point(float x,float y)

Given points : S,E,L,R

These points should make 4 arcs of W smooth. Otherwise,it is bad input.

Variables calculated in in problem 1

Both Circle A and Circle B are tangent to Circle S and cir Circle E

CircleCenterL : The center point of common tangent CircleA

CircleCenterR : The center point of common tangent CircleB

RadiusL : the radius of circle A

RadiusR: the radius of circle B

## Objectives

The medial Axis of caplet W

## Output

An exact medial axis of W will be visualized.

It will be represented in a point set of Ellipse or Hyperbola.

Ellipse(float a, float b,float c)

Hyperbola(float a,float b,float c)

## Validation

Take a point on the medial axis and calculate the closest distance between this point and two border arcs. If the results are same, it is a valid point on medial axis.

Use this method to examine all the points in the point set used to represent this medial axis. If all of the points are valid, this solution is correct.

## Outline of your approach

The medial axis in this problem is a Ellipse or Hyperbola. For easier calculation, I rotate it and make the foci on the X - Axis. Then calculate it and get some points we need to draw the medial axis. Because the points are in a rotated coordinate, I need to rotate them again into the original coordinate. Finally, I can draw the medial axis with the points I get.

## Results

When the four arcs of caplet W is smoothly connected(correct input), my approach will work well and generate the medial axis.

If the input is bad, then the approach will fail because we can’t even find four smoothly connected arcs to make caplet W.

## Details

There are two conditions of medial axis in this problem: Ellipse and Hyperbola

Justification:

Assume Point T is on the medial axis

D is the distance between T and arc A or arc B.

D1 is the distance between T and circle center A of border arc A.

D2 is the distance between T and circle center B of border arc B.

RadiusL is the radius of circle A

RadiusR is the radius of circle B

Three different relations between circle A and circle B

1. If circle A and circle B don’t have any overlap, then the medial axis is outside the two circles

D1 = D + RadiusL;

D2 = D + RadiusR;

D2 - D1 = RadiusR - RadiusL.

Or

D1 = D - RadiusL;

D2 = D - RadiusR;

D2 - D1 = RadiusL - RadiusR.

Since RadiusL and RadiusR are given,D2 - D1 is a fixed value,so the medial axis is on a Hyperbola

2.If circle A and circle B have overlap but a circle is not completely in another, then there will be two conditions

First :

D1 = RadiusL - D;

D2 = RadiusR - D;

D2 - D1 = RadiusR - RadiusL.

Since RadiusL and RadiusR are given,D2 - D1 is a fixed value,so the medial axis is on a Hyperbola

Second

D1 = RadiusL - D;

D2 = RadiusR +D;

Or

D1 = RadiusL + D;

D2 = RadiusR - D;

D2 + D1 = RadiusR + RadiusL.

Since RadiusL and RadiusR are given,D2 + D1 is a fixed value,so the medial axis is on a Ellipse

1. If circle A and circle B have overlap and a circle is completely in another circle

D1 = RadiusL - D;

D2 = RadiusR +D;

Or

D1 = RadiusL + D;

D2 = RadiusR - D;

D2 + D1 = RadiusR + RadiusL.

Since RadiusL and RadiusR are given,D2 + D1 is a fixed value,so the medial axis is on a Ellipse

Pseudocode:

CL ← Circle center of common tangent circle A

CR ← Circle center of common tangent circle B

S ← Circle center of circle S

E ← Circle center of circle E

RadiusL ← radius of common tangent circle A

RadiusR ←radius of common tangent circle B

Mat ← A matrix that translate the foci(CL, CR) of curve to the X-axis

Mat\_inv ← inverse of Mat

C = distance(A,B) / 2

St = Apply(S, Mat)

Et = Apply(E, Mat)

SAEllipse = distance(St,CL) + distance(St,CR)

EAEllipse = distance(Et,CL) + distance(Et,CR)

SAHyperbola = abs(distance(St,CL) - distance(St,CR))

EAHyperbola = abs(distance(Et,CL) - distance(Et,CR))

Ellipse\_2a = RadiusL + RadiusR

Hyperbola\_2a = abs(RadiusL - Radius)

**If** SAEllipse == Ellipse\_2a && EAEllipse == Ellipse **then**

A = Ellipse\_2a / 2

B = A\*A - C\*C

P = points in Ellipse(A,B,C)

**else if** SAHyperbola == Hyperbola\_2a && EAHyperbola == Hyperbola\_2a

A = Hyperbola\_2a / 2

B = C\*C - A\* A

P = points in Hyperbola(A,B,C)

**endif**

All points in P apply Mat\_inv

Draw (P)

## References

## Further research

The algorithm is useful only for circles and arcs. We can do more research on how to create a medial axis of irregular figure.

# References & resources

*List papers, books, courses, software that may be useful.*